An Approach Using Linear Regression

Divide equation (3) by Y to get

$$\frac{1}{Y}\frac{dY}{dt} = r - \frac{r}{K}Y. \tag{5}$$

Approximate $Y^{-1}\dot{Y}$ by the average rate of change between observations divided by the average population between observations,

$$\left(\frac{1}{Y}\frac{dY}{dt}\right)_i \approx \frac{\frac{Y_{i+1}-Y_i}{t_{i+1}-t_i}}{\frac{1}{2}(Y_{i+1}+Y_i)} = 2\frac{Y_{i+1}-Y_i}{(Y_{i+1}+Y_i)(t_{i+1}-t_i)}.$$

This gives a relationship between the right-hand-side of (5) and your data. To be consistent Y in the left-hand-side should also be approximated using the average $\frac{1}{2}(Y_{i+1}+Y_i)$. Now, to get an estimate for parameters r and K one should regress the data

$$\left\{\left(\frac{(Y_{i+1}+Y_i)}{2},\ 2\frac{Y_{i+1}-Y_i}{(Y_{i+1}+Y_i)(t_{i+1}-t_i)}\right)\right\}_{i=1}^{N}$$

against the line

$$r - \frac{r}{K}Y$$

to determine r and $\frac{r}{K}$.

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