A Useful Simplification

This model, unfortunately, is plenty complicated, and involves two quantities which are very difficult to measure (g_i and a). One avenue of simplification is to assume that certain effects are not going to be important (for example, if one assumes that osmotic leakage is small, then essentially $\gamma = 0$). An additional simplifcation arises from the fact that equations (6 - 8) form a *compartment model* if $\omega = 0$, or alcohol inhibition is neglected. That is, at some level everything is accounted for, nothing lost, and therefore the derivative of some summed up quantity must be zero. To see this, set $\omega = 0$ and add up (6 - 8) in the following way:

$$\begin{array}{rcl} \dot{g}_o & = & -\alpha\ g_o y + \gamma\ (g_i - g_o) y \\ \dot{g}_i & = & +\alpha\ g_o y - \gamma\ (g_i - g_o) y - \beta\ g_i y \\ +\ \dfrac{\beta}{\epsilon}\dot{y} & = & \qquad\qquad\qquad\qquad +\dfrac{\beta}{\epsilon}\epsilon\ y g_i \\ \hline \dfrac{d}{dt}\left(g_o + g_i + \dfrac{\beta}{\epsilon} y\right) & = & 0, \end{array}$$

and consequently we deduce

$$\frac{d}{dt}\left(g_o + g_i + \frac{\beta}{\epsilon} y\right) = 0,$$

or

$$g_o + g_i + \frac{\beta}{\epsilon} y = Constant.$$

This allows us to eliminate one of the unmeasurable variables, g_i, through the relation

$$g_i = C - g_o - \frac{\beta}{\epsilon} y.$$

This results in coupled equations for y and g_o when the above expression for g_i is substituted into (6) and (8):

$$\dot{g}_o = -\alpha\ g_o y + \gamma\ y \left[C - 2g_o - \frac{\beta}{\epsilon} y\right], \qquad (10)$$

$$\dot{y} = \epsilon\ y \left[C - g_o - \frac{\beta}{\epsilon} y\right]. \qquad (11)$$

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2000-07-31